Can Curriculum Materials Support Teachers' Learning? Two Fourth-Grade Teachers' Use of a New Mathematics Text

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Abstract

Current wisdom suggests that accomplishing the changes reformers call for in mathematics teaching will require significant learning on the part of teachers. The aim of this study was to examine whether and how a reform-oriented textbook could contribute to such learning. Using case study methods, I analyzed the nature and contexts of 2 fourth-grade teachers' learning during their first year of using a new mathematics textbook. Analyses across the cases revealed key activities involved in teaching and using texts that created opportunities for learning. These activities included analyzing students and mathematical tasks and making decisions about how to proceed, suggesting that materials most likely to foster teacher learning are those that engage teachers in these processes.

For over a decade, mathematics education in the United States has been in the midst of reform. In 1989, the National Council of Teachers of Mathematics (NCTM) published *Curriculum and Evaluation Standards,* which called for increased emphasis on mathematical reasoning, understanding, and problem solving and decreased emphasis on memorizing rules and computational procedures. The response was a flurry of activity at the national, state, and local levels aimed at revamping the content and pedagogy of mathematics instruction. One of the first steps many school districts took in response to these calls for change was to adopt a new textbook, a reasonable approach given the substantial place textbooks have in U.S. classrooms (Tyson-Bernstein & Woodward, 1991).

Research on teaching, however, has raised questions about the power of revised textbooks to foster the changes required by these reforms. The failure of the teacher-

proof curriculum materials of the 1950s and 1960s to facilitate substantial curricular change suggests that teachers, rather than texts, ultimately determine what is taught (Sarason, 1982). Recent research on the use of curriculum materials has indicated that teachers' beliefs and knowledge about teaching, learning, and the subject matter influence their decisions about what and how to teach more than what is presented in texts (Putnam, 1992; Remillard, 1992; Stephens, 1982). In addition to raising doubt about the power of textbooks to foster change, these findings suggest that, as in achieving any new purpose, changing instruction will require learning on the part of educators (Ball, 1997; Cohen & Barnes, 1993).

Recognizing the need for substantial teacher learning in the process of pedagogical change, many involved in recent reform efforts have been cautious about relying too heavily on curriculum materials, choosing instead to emphasize teacher development (Nelson, 1995). A number of successful programs to foster teacher learning have been developed and described in the literature (e.g., Barnett & Friedman, 1997; Campbell & White, 1997; Franke, Fennema, & Carpenter, 1997; Schifter, 1998; Stocks & Schofield, 1997). Some studies of teacher development projects, however, have identified a need for teachers in the midst of pedagogical change to have well-designed curricular guidance, noting that teachers who have begun to think about mathematics teaching and learning differently are likely to struggle with how to use these ideas in their classrooms (Brown, Smith, & Stein, 1996).

In the study reported in this article I explored this tension between the potential and limits of reformed curriculum materials. The research questions guiding the study were: Can using a reform-oriented textbook contribute to teachers' learning that is related to changes in their mathematics teaching? And, if so, how? Using case study methods, I examined the nature and contexts of two teachers' learning during a year in which they used a revised mathematics textbook for the first time. I explored when, how, and what the teachers learned, examining the factors that seemed to contribute to the learning I observed. These factors included, but were not limited to, the textbook. Through a cross-case analysis of their learning and the ways the text did and did not contribute, I considered how reform-oriented curriculum guides might support teacher learning. I also considered implications that these findings have for the selection and use of reform-based curriculum materials.

The Importance of Teacher Learning

This study was based on the assumption that a close relation exists between teacher learning and pedagogical change. Teachers draw on their knowledge of and ideas about mathematics and how it is learned in making pedagogical decisions (Thompson, 1984). Thus, accomplishing pedagogical change is likely to require opportunities for teachers to examine and expand their mathematical and pedagogical understandings (Schifter, 1998) as well as to unlearn many of their old practices and learn new ones (Ball, 1997; Cohen & Barnes, 1993; Heaton, 1994). The need for new learning is particularly severe in the case of current reform efforts in mathematics education because many of the ideas central to these efforts are foreign to teachers. Teachers are being asked to teach in ways that are unfamiliar to them, ways that they did not experience as students. Having grown up in the same educational system they are now being asked to change, few teachers possess the mathematical tools or mental images to accomplish such shifts in their teaching (Cohen, 1989). Furthermore, the reforms themselves are not prescriptive. Rather, teachers are encouraged to build on students' ideas and mathematical knowledge to help them develop conceptual understanding and relevant mathematical skills. Engaging students in this

kind of mathematical learning requires most teachers to think differently about mathematics and the process of learning it (Heaton, 1994; Schifter, 1998).

Reports from professional development projects have provided considerable evidence that the kind of learning that leads to fundamental change in teaching occurs over a long period of time, with extensive support and multiple opportunities to experiment and reflect (Loucks-Horsley, 1997). Teachers also benefit from participating in professional communities (Secada & Adajian, 1997; Stein & Brown, 1997) as well as engaging in mathematical activities themselves, talking with others about their learning, and examining students' thinking (Franke et al., 1997; Schifter, 1998; Smith & Featherstone, 1996). This research has focused on the nature of the professional development activities and related teacher learning. If involved at all, reformed curriculum materials have been treated as a backdrop or contextual factor (e.g., Stocks & Schofield, 1997). The question underlying my study was whether curriculum resources for teachers could foster the type of support, reflection, and examination of mathematics and students' thinking currently understood to be critical to teacher learning.

Method

In order to examine how curriculum resources might provide opportunities for teacher learning, I observed and analyzed two fourth-grade teachers who were using a newly adopted mathematics textbook for 1 school year (1992–1993). The analysis focused on the learning that occurred and the pertinent characteristics of the text, the teacher, and the teaching context. I viewed learning as changes observed (*a*) within the individual teachers (Cobb, 1995), such as in their mathematical understandings and their ideas about teaching and learning; as well as (*b*) in their practices, that is, the tools they used to make and act on pedagogical decisions (Wertsch, 1995).

Sample Selection

District. The teachers taught in different schools in the Maplewood District (all names are pseudonyms), situated in a mid-sized, predominantly working-class city in the Midwestern United States. The district's student population consisted of approximately 50% Anglo-American, 35% African-American, 10% Latino, and 5% Asian-American. I selected this district because it had recently adopted a new mathematics textbook in order to align instruction more closely to current national and state reforms.

Teachers and schools. Both schools served students of low- to middle-income families and were similar in size. Although class size changed during the year, both teachers had approximately 25 students in their classes. The teachers were Anglo-American women who had spent most of their approximately 30 years of teaching in the Maplewood District. Nevertheless, they had contrasting ideas about teaching mathematics, and they worked in school environments that offered radically different types of support and opportunities.

Jackie Yarnell taught at Kipling Elementary School, which had recently become a professional development school (PDS) associated with a nearby university school of education. This association made professional growth a central focus of the school. Jackie was an active participant in several PDS activities, one of which related to math. As a result, she had seen unconventional teaching practices, experimented in her teaching, and talked with others about her observations and reactions. In a short time, she had reexamined many of her beliefs about teaching, learning, and mathematics and become determined to provide students with opportunities to think, invent solutions to problems, and discuss their ideas with others. Jackie was also participating in an experimental project that involved having the same students from third to fifth grade. Thus, at the beginning of fourth grade she knew her students well and had

already established a collaborative community in her classroom.

Catherine McKeen taught at McKinley Elementary School. McKinley did not focus on professional development. The school provided few formal opportunities for teachers to interact around teaching issues, and Catherine participated in few professional development activities. She had well-developed and fairly conventional ideas about what learning math included. Computational mastery was at the core of her instructional goals, which she pursued through giving students careful guidance and repeated practice. Through conversations with colleagues and participation in district activities, Catherine had recently become aware that learning math included more than computation, that students needed to learn to think and to apply computational knowledge. She found these ideas compelling and wanted to incorporate them into her teaching, but not at the expense of computational mastery. Like many teachers in this position, she saw the new textbook as a tool to help her make the necessary changes in her teaching.

The differences between the teachers and their school settings allowed me to examine how these characteristics affected their use of the new textbook. By holding the district, grade level, amount of teaching experience, and textbook constant, I was able to focus on the different ways the teachers' ideas and understandings, their students, and their school contexts contributed to their learning from the textbook.

Textbook

The reform-oriented textbook that the Maplewood District had selected was *Mathematics Plus*, published by Harcourt Brace Jovanovich (HBJ) (Burton et al., 1992). I refer to it as "reform oriented" because it was one of many commercially published texts revised to reflect changes called for by the NCTM *Standards*. Although the extent to which the publishers understood and were committed to reform can be questioned, the HBJ text was similar to most commercially published texts available in 1992 (at that time, a number of noncommercial curriculum materials were under development, but few were available).

In using the term "textbook," I am referring to the commercially published curriculum presented in the teacher's guide. In the case of the HBJ textbook, this includes a copy of the student textbook as well as a number of additional suggestions for the teacher. Some of the suggestions are designed to guide the teacher in using the tasks in the student text. Others, however, are additional tasks and activities designed to complement those in the student text.

In many ways, the text's offerings reflected a notable change from texts of the past. According to both teachers, the HBJ text was appealing because of its emphasis on problem solving. In addition to daily lessons, it provided a nonroutine "problem of the day" for students. The pages of the teacher's guide were packed with activities and teaching suggestions, many of which involved manipulatives, calculators, or student collaboration.

In other ways, the HBJ text did not reflect a significant change. Like texts of previous years, it presented the curriculum in 13 familiar chapters, each consisting of a collection of daily lessons, and made few connections among lessons or chapters. In spite of the new elements added to text, much of the work on the pages of the student textbook was devoted to developing procedural skills. The guidance in the teacher's manual consisted of descriptions of tasks for students and, in some cases, brief steps to follow in presenting them.

Data Collection and Analysis

Classroom observations and interviews formed the corpus of the data. During three rounds of data collection (each ranging from 2 to 4 weeks) distributed across the school year, I observed, took field notes, collected documents, and interviewed the teachers about their mathematics teaching

and how they used the textbook. I also conducted a baseline interview and an end-of-year interview with both teachers to gather background information and to assess changes in their beliefs and understandings.

Analysis involved developing interpretive cases of the teachers' mathematics teaching, textbook use, and learning that were descriptive, explanatory, and analytic. Even though cases do not support generalizations across situations or teachers, they suggest themes and highlight issues or concerns (see Stake, 1978; Yin, 1994). I drew on descriptive data to characterize each teacher's thoughts about textbook use and then used that characterization to explain her interactions with the textbook and its effects on her thinking and teaching. I followed the writing of the two cases with a cross-case analysis in which I examined similar patterns in both. This analytical process involved developing themes and returning to each case to check their validity. The findings presented here resulted from the cross-case analysis of the teachers' learning as they used the new text.

Analytical Frame

My analysis was guided by a theoretical frame offered by research on what Doyle (1993) has called teachers' "curriculum processes," that is, the processes through which teachers construct or enact curriculum. Here, curriculum is not what is written in textbooks or in policy guidelines; it is what actually takes place in the classroom. To examine teachers' learning through using a text, I examined the role the text played in their curriculum processes and related changes in their thinking and practices.

Teachers' curriculum processes include reading and translating curricular ideals written by others into ideals that teachers intend and enact in the classroom. My analysis examined both the planning and enacting components of the teachers' curriculum processes in order to assess the potential for learning in each. This analysis also resulted in a clearer articulation of how teachers construct curriculum through their teaching (Remillard, 1999).

Results

Analyses of the two teachers' curriculum processes and related changes in their mathematical understandings, ideas about teaching and learning, and their pedagogical practices suggested that both teachers learned a good deal over the year. Moreover, this learning appeared to be related to a set of activities central to constructing curriculum. These activities all involved a form of *reading*. Generally, reading refers to a constructive and dynamic process of meaning making through engaging written text (Pearson & Stephens, 1994; Rosenblatt, 1994), such as would be found in a textbook. My analysis of the teachers' construction of curriculum indicated, however, that they "read" more than the textbook. In order to make curricular decisions, the teachers read students and the tasks they engaged in, as well as the suggestions in the textbook. These interactive reading processes seemed to be at the heart of both teachers' learning.

In the following sections I describe the three types of reading I observed and the role they played in Catherine's and Jackie's learning. The three types overlap and interact in actual practice. That is, the teachers rarely engaged in one type of reading in isolation. I have separated them only for analytical purposes. I also examine possible missed opportunities for learning. My goal is to consider both the potential and limits of curriculum materials in contributing to teacher learning. As I discuss later, my analysis suggested that well-designed curriculum materials can play a role in supporting change in teachers' thinking and practices, but not in ways most commonly imagined by those who design and select these materials.

Learning through Reading the Text

When selecting and designing tasks to present to students, both teachers read the

HBJ textbook. The way they read the text, however, differed as much as the tasks they chose. Their reading was selective and interpretive. They read different parts of the text and drew on their own perspectives to make meaning of what they read. The differences in their reading led to differential opportunities for teacher learning.

Learning through appropriating tasks. In designing her lessons, Catherine selected tasks from the teacher's guide and inserted them into her teaching. She viewed the reform agenda as an attempt to add additional topics, such as problem solving and other noncomputational skills, to the existing mathematics curriculum, and she saw the text as a source of new mathematical tasks to draw on. Thus, using the text involved a process of selecting tasks. Her intent was to use the tasks as they were presented in the text. But, as is always the case, using any task offered in a text in a classroom requires making ongoing adjustments. It was this process of adjustment, which involved examining students' encounters with the tasks and adapting tasks, that prompted Catherine to revise many of her views about the nature of mathematics, her mathematical understandings, and practices.

One example of Catherine's learning through using tasks in the text involved the set of nonroutine problems titled "problem of the day." Each day Catherine posted one problem from this collection for students to work on when they arrived in the morning. She generally gave them about 5 minutes to work alone and then collected their papers before "going over" the solution with the class. That is, she showed the students the method she had used, often translating the problem into a computational algorithm.

Although she used them regularly, Catherine initially voiced many concerns about these nonroutine problems. They were unwieldy, not like the routine story problems with which she was familiar. Many of the problems of the day, she pointed out, involved multiple steps and required several operations. Some problems did not even involve numbers. "You can't even put a mathematical sentence to it," she explained, pointing to one of the problems she had given her students in November:

> Mr. Sanchez and Ira are waiting in line. Ira is the fifth person in line. Mr. Sanchez is the last person. There are two more people behind Ira than there are in front of him. How many people are in front of Mr. Sanchez? (Interview, Nov. 19, 1992)

She admitted that she was not sure how she would explain her own intuitive solution to students.

Catherine found these problems particularly unsettling because most of her students were not familiar with them and frequently did not know how to solve them. "They are not familiar with problems that have three steps in them," she explained after only two students had successfully solved that day's problem (Interview, Dec. 17, 1992). She found it frustrating that the teacher's guide did not suggest how to help students approach these problems.

Because she wanted to give the new textbook a "fair try," Catherine assigned her students one of these problems daily. She hoped experience would help more students learn to solve them. Occasionally, she put aside other morning activities and worked with students to figure out each step of the problem.

Changes in Catherine's attitude toward the problems of the day emerged about halfway into the school year. She expressed less doubt about the appropriateness of the problems and spoke adamantly about the need for students to be able to solve a range of problems: "Now, problem solving has become very important to me," she remarked during one of our conversations (Interview, March 11, 1993). Catherine attributed her change in perspective to (*a*) her attempts to help students develop solutions, (*b*) her observations of students who were unable to solve the problems, even when she allowed them to use calculators,

and (*c*) the unanticipated solutions students described to her.

Walking students through the process of selecting and using a strategy and then thinking about whether their results were logical increased Catherine's respect for the type of thinking these problems required. The fact that students were unable to think through this process by themselves demonstrated to the teacher the value of "problem-solving" skills. "The more they're working on the problem solving, the more important these kinds of problems are becoming to me," she explained (Interview, April 3, 1993).

Catherine's students' successful work on these problems also contributed to her learning. As more students gained confidence in their own problem-solving abilities, she found that some students began to offer their solutions to the class, revealing to her the multiple ways most problems could be solved and her students' ability to invent solutions: "When we would do the problem of the day, they would do it, and we'd talk about it. And then I'd show them, you know, how we got the answer that we got on the board, and inevitably I'd have two or three kids say, 'This is what I did.' And they would have a different way of doing that and then some children would do an equation, and some people would draw a picture" (Interview, June 25, 1993). Impressed by students' varied approaches, Catherine eventually built opportunities for students to present their strategies into the daily discussion of the problem of the day. She continued to walk them through her approach to solving the problem, but she would always follow with: "Did anyone solve it a different way?" Seeing students' differing approaches challenged Catherine's view that each problem had one, best solution path that students needed to be taught: "When I grew up, it was always just a certain way you had to do it. That's what we were led to believe. We had to do it the way the teacher did it. . . . Now I think it's neat how children can try their own way" (Interview, April 3, 1993).

Catherine ended the school year having undergone a considerable shift in her view of problem solving and its importance and her perception of her own and her students' abilities to solve problems. In our conversations late in the year, she characterized problem solving as critical for students to learn: "I always thought of [problem solving] as games, fun-time games, you know. . . . I see the reason for these things now, even though there may not be mathematical words involved or a mathematical equation to solve; it still, it's a thought process that's important in math" (Interview, June 25, 1993).

These changes in Catherine's views about the nature of mathematics and mathematical understanding influenced her future reading of the text and the tasks she used with students. Because she began to value mathematical understanding, she paid more attention to the tasks in the teacher's guide that were designed to nurture understanding. For example, in February she tried the suggestion in the textbook to use base-10 blocks to model division. Prior to this, her approach to teaching computation was solely procedural and symbolic. Similarly, during the fraction unit that she taught in April, she used many of the exploratory tasks in the text that emphasized underlying meanings. Catherine had routinely ignored these tasks that focused on underlying concepts in the fall, choosing to devote more time to procedural exercises on the pages of the student text.

Catherine's learning with respect to these tasks was similar to her learning from using problems of the day. She often began skeptically, sometimes using an approach because the teacher's manual suggested it, not because she was convinced of its efficacy. The process of engaging in the task with students, examining and responding to their encounters with it, often sparked her to reexamine her initial assumptions.

For example, the success she had using the base-10 blocks to model division prompted her to reconsider her views on using manipulative materials. In the fall she explained that she did not think her students "needed" to use them. Later, she admitted that attempts to use them in previous years had resulted in disorder and confusion for the students and frustration for her. She recalled that using manipulatives tended to leave her "really rattled" (March 11, 1993). This did not happen when she followed the suggestions in the new teacher's guide to model "bringing down" the next digit of the dividend in the division algorithm. "It was really interesting. . . . You have a long 10 and three little blocks. So they break these up and they take 10 of the individuals to show how they got 13 . . . and they understood!" (Feb. 16, 1993). She reported this incident as her first successful attempt at using manipulatives.

Learning through inventing tasks. Unlike Catherine, Jackie did not select tasks from the text. She used the text as a source of mathematical and representational ideas from which she adapted and invented her own tasks. These ideas ranged from mathematical concepts or relationships underlying the tasks in the text to ways to model or explore these ideas. Because she believed that the reforms were aimed at developing students' understandings of mathematical ideas and relationships, she rejected tasks that required only rote skills. However, if she believed that an idea underlying a task was valuable, she invented a new task that focused on this idea. The tasks she designed engaged students in exploring mathematical ideas, discussing their understandings with one another, and developing their own solutions to problems.

Jackie's approach to reading the text and inventing tasks led to teacher learning that was substantially different from Catherine's. The process of thinking through the mathematics in order to invent tasks deepened Jackie's understanding of central mathematical ideas in the curriculum and their relations to one another. For example, in September she believed that place value and column addition and subtraction were related yet distinct topics. The first was a precursor to the latter. If they understood place value, "[addition and subtraction] would come more easily" (Interview, Sept. 24, 1992). Thus, she began the school year with the chapter on place value and planned to complete it before moving on to column addition and subtraction. Her process of inventing tasks led her to see and treat these two topics as one.

The following example from early in the fall illustrates the way Jackie read the text and invented tasks throughout the year and the effect these activities had on her ideas about mathematics. She began by reviewing the explanations, examples, and tasks on the page in the student text and concluded that the central idea was the relation between the values of the places in the base-10 system (e.g., that 140 hundreds was another way to show 14,000). She believed it was crucial that students understand this idea but that the tasks on the page were unhelpful because they merely asked students to show these equivalencies by filling in blanks such as 300 = ____ tens. "[The book] gave some numbers like a thousand nine hundred and said, 'show four ways of writing it.' But I really felt that it was patterning, you know, they would just follow the pattern. I was hoping that they would start looking at it and thinking what a hundred is" (Interview, Sept. 16, 1992). With this goal in mind, Jackie invented a task that required students to use drawings of base-10 blocks to illustrate and "prove" the relations written in the student text, showing, for example, how 300 could be represented as 30 tens or 20 tens and 100 ones.

Thinking through the mathematical ideas underlying the base-10 number system in order to design tasks that would help students understand place value prompted Jackie to strengthen her own understanding of place value and its relation to computation. As she developed tasks that engaged

students in exploring the multiple ways a number could be represented as well as the relations among these representations, she realized the fundamental connections between place value and regrouping in column addition and subtraction. The importance of this connection was made even stronger for Jackie when a student excitedly demonstrated to the class that 24,000 could be represented as 2 ten thousands and 4 thousands or 24 thousands. As she put it, "It kind of hit me that this was a logical jump to regrouping" (Interview, Oct. 15, 1992). Following this realization, Jackie began to invent tasks that integrated computation and place value in a way that was new to her. Through this work, she hoped her students would "actually see what carrying—what you were doing when we say carrying, you know, regrouping—is" (Interview, Oct. 15, 1992).

Jackie's deeper understanding of the role that place value played in column addition prompted her to examine place value in other operations. She began to see place value as a powerful idea underlying all computational processes. This understanding was evident in her response to my question about what she intended to pursue after column addition and subtraction: "I'm hoping that from there we could try making sense out of multiplication, which is funny because I never thought of what multiplication was too much until I started doing this. And I'm trying to think, 'Well, how am I going to make sense of this?' " (Interview, Oct. 30, 1992). As I questioned her further, she described a new realization about multiplication problems such as 43×51. She had always taught her students to multiply 1×3 and then 1×4. Now, she realized that "it is really 1×40," and she wanted her students to understand this as well. "So they're actually seeing that it's not 1×4; it's 1×40" (Interview, Oct. 30, 1992).

Missed opportunities for learning. Although both teachers learned from reading the textbook, the fact that what and how they read was shaped by their views of mathematics and their understanding of the reforms restricted their opportunities to examine new mathematical ideas. In order to learn from using tasks, Catherine had to be willing to use them in her classroom. Similarly, if Jackie was to learn about a mathematical idea, she needed to see the idea in the text and then be convinced that it was valuable. This did not always happen.

Catherine's views about mathematics and the reforms led her to dismiss certain chapters in the text. She held firm ideas about the importance of computation and how it should be taught, and she believed that the reforms did not address computation. As a result, Catherine tended not to read the chapters on computation but used her own approach to teaching these topics. Thus, she did not use tasks from these chapters or permit alternative ideas about teaching computation to enter her practice for the first 5 months of the school year. As I discussed earlier, changes in her views of mathematics eventually led her to try a suggestion for using base-10 blocks to teach division in February. This was the first and only time that I observed her using unconventional tasks from the text to guide her teaching of computation. Following this instance, she used conventional approaches to teaching students the long-division algorithm.

Jackie's missed opportunities were also a function of how she read the text. Though she focused her reading on underlying concepts, her views of the mathematics content shaped the meanings she made of them. Because she believed it was important, she examined the structure of the base-10 place value system more deeply and invented tasks to help students do the same. She saw these ideas as central to understanding column addition and subtraction. In contrast, she did not necessarily read or value other important ideas in the text. For example, the HBJ text made estimation prominent in instruction on the four operations, devoting at least one lesson to estimating prior to developing a particular algorithm. Although

not explicitly stated in the textbook, the idea underlying this approach seems to be that understanding the meaning of each operation developed through estimation should precede learning the actual procedure. Jackie did not see estimation in this way and thus discarded these estimation tasks. She explained that she did not want to "confuse them" by introducing estimation before students had learned the actual computational steps. She said: "I thought before they're ready to estimate, let's actually do some [addition]" (Interview, Oct. 22, 1992).

In this instance, Jackie's orientation to textbook use allowed her to ignore the text's efforts to present estimation as something other than an isolated skill used to check for computational accuracy. Her view of estimation as separate from computation was unaffected by the text. Thus, her approach to reading the text sometimes led to missed opportunities as well as opportunities to learn.

Learning through Reading Students

Whether tasks were appropriated from the textbook or invented by the teacher, using them in the classroom involved the teachers in sustaining students' work on them. Doing so involved checking on students and reading their progress with respect to each teacher's goals and her ideas about what it meant to know math. For example, because Jackie attended to students' understandings of specific mathematical ideas, she carefully noted their explanations for their solutions in order to plan how to challenge them further. Catherine's focus, particularly at the beginning of the year, on helping students be "successful" in math without becoming too frustrated led her to pay attention to what they were doing and whether it was likely to lead them to the correct answer. Later in the year, Catherine attended to what students understood and were thinking, sometimes spontaneously veering from suggestions in the text to probe students' responses.

The teachers relied on their reading of students, particularly when students were struggling. As a result, they found themselves paying close attention to students' thinking in order to help them. Through this process, the teachers often examined mathematical ideas more closely and reconsidered their own views of mathematics and of students' abilities.

Catherine's efforts to look beneath her students' troubles with the problems of the day, described earlier, led her to develop a sense of and appreciation for the mathematical abilities the problems required. Similar efforts to make sense of and respond to student confusion in other areas also prompted her learning about mathematics and students. For example, during the fraction unit she taught in April, students had difficulty using counters to solve problems such as $\frac{3}{4}$ of 12. In order to make sense of their struggles, Catherine examined the process and her students' errors and speculated that they did not "understand that this [pointing to the symbolic notation] is the same as this [pointing to the representation with counters]" (Interview, April 8, 1993). Concluding that they needed a firmer understanding of the meaning of the numerator and denominator, she designed the next lesson to direct students' attention to this relation. This emphasis on connecting the symbolic notation to the concrete representation was markedly different from the view she articulated earlier in the year. In the fall she argued that students "understood" when they were able to complete the tasks she gave them accurately, even when the tasks merely involved symbolic manipulations. Using manipulatives as prescribed in the teacher's guide provided her an opportunity to examine students' understandings of the relation between symbolic and concrete representations, causing her to question her assumptions. Reading and responding to students' struggles with the conceptually oriented tasks broadened her view of the nature of mathematical understanding and increased her understanding of her students' mathemati-

cal abilities. As she concluded: "I think that people that really are able to do problem solving and the higher mathematics are the ones that understand the connections. And then the people, like myself, that just learned the processes when I was younger, had more trouble with the higher math because I really didn't understand a lot of the reasons why certain things worked" (Interview, April 8, 1993).

These changes in Catherine's thinking were evident in the way she reflected on her decisions as a teacher. In April she showed me a problem she had devised that asked students to compare $\frac{3}{4}$ and $\frac{6}{8}$ of 24 using a realistic situation. She explained, "If I had been really thinking, I would have asked them how they knew or why they gave the answer they did." She went on to explain that they had been working on equivalent fractions and that asking them why would "be making them put into their words" what they understood about the relation between the two fractions (Interview, April 14, 1993).

Jackie's efforts to read her students also stimulated her learning. Helping students through struggles with fractions led her to explore her own understanding and her students' thinking more carefully. The following lesson from a unit in April illustrates this learning process. On the basis of introductory activities Jackie had used the 2 previous days, she believed that students understood some basic ideas about fractions. Wanting them to apply their knowledge, she gave each group of four students a chocolate bar composed of 12 segments and asked them to indicate whether they wanted one-third, one-fourth, or one-sixth of the candy bar. The students' work on this task revealed a variety of misconceptions about fractions. In particular, Jackie noted that students had trouble differentiating between thirds, fourths, and sixths of 12 because each fractional part involved a certain number of pieces of chocolate. Jackie later explained that listening to her students revealed how little they actually understood about fractions: "I figured a kid understood what a fourth was, but what they were understanding was a fourth only if something is divided into four equal pieces, without knowing that those pieces might still have parts of something in it" (Interview, April 12, 1993). To reach this conclusion, Jackie had to examine her own understanding of equivalent fractions, looking more deeply at the concepts underlying the rules that she thought she understood: "Things are dawning on me. . . . I never thought that . . . one-half was the same as five-tenths. I mean, I did equivalent fractions because I had to, but I don't think it really dawned on my mind what that meant. . . . It's actually making more sense" (Interview, April 12, 1993). This deeper understanding led her to construct a set of lessons on the meaning of the numerator and denominator in discrete fractions.

Learning from Reading Tasks

In addition to reading students, both teachers continually examined and analyzed the tasks students were engaged in. This reading was sometimes aimed at helping students with a task that caused them difficulty. At other times it was prompted by students' unanticipated approaches or comments, which required a response from the teacher. Both teachers' reading emphasized the mathematical demands and goals of a task. This process of reading mathematical tasks as students engaged in them prompted the teachers to learn more about mathematics and about students' abilities.

To help students solve some of the most complex problems of the day, Catherine spent a great deal of time analyzing the problems herself. She considered what type of thinking they involved, how she would do them herself, and how she would articulate the intuitive approaches she used. As I described previously, Catherine's efforts to analyze these tasks enabled her to see their mathematical value and prompted changes in her view of her students' abilities.

As the year progressed, Catherine used more tasks from the text that were designed to focus on underlying mathematical meanings. All but one of the seven fraction lessons I observed her teach in the spring involved the students in folding, shading, and cutting paper to represent and compare fractional parts and in using counters to model discrete fractions. As she led students through these tasks, Catherine analyzed their mathematical significance. She was struck by the mathematical demands of the tasks and their potential to build student understanding. "I've done the fraction pies and things like that. Usually the way I do it is draw on the board. With this [referring to the suggestions she used in the textbook] *they* [the students] have to do the manipulating. I think it means more to them than the drawings on the board" (Interview, April 8, 1993).

Jackie reexamined the tasks her students worked on primarily to interpret and respond to their thinking. As I described earlier, she had designed most of the tasks she used herself. However, sometimes her students' work on a task was so unexpected that she reexamined it for insights on how to proceed. For example, analyzing and responding to students' work on the following combination problem pushed her to explore the nature of the problem as well as the mathematical thinking it required. The problem asked students to consider the possible scores a child would get if she threw three darts at a dart board containing three scoring rings: one worth five points, one worth two points, and the outer ring worth one. Watching students work, Jackie noted that few approached the problem systematically. In addition, they recorded solutions such as 2-1-2 and 2-2-1 as two different possibilities, although both totaled 5. These haphazard approaches prompted Jackie to compare what the task asked students to do with how they were approaching it. She realized that if she pushed her students to examine the validity of their answers, the task could do more than provide them practice with addition. It could "push them to think a little bit more" so they would "see that order didn't make a difference" (Interview, Oct. 15, 1992). Thus, she began the class discussion by posing the question, "How do you know you have them all?" (Observation, Oct. 15, 1992). Jackie had not selected this problem for these reasons. The value of developing systematic approaches to such problems became apparent to her as she analyzed the problem while students worked on it.

Discussion

The previous descriptions of the two teachers' reading and related learning provide examples of three types of learning—the nature of mathematics, mathematics content, and students. This learning appeared to influence their subsequent reading and decision making.

The types of mathematical learning both teachers experienced are similar to the types of learning that Russell et al. (1995) have argued are critical to teachers' efforts to improve their mathematics teaching. In their work with teachers, they noted three kinds of explorations that teachers engage in that can lead to learning about mathematics: (*a*) exploring the content in preparation to teach, (*b*) examining ideas underlying student confusion, and (*c*) engaging in mathematical thought with students.

Jackie's approach to inventing tasks, for example, led her to examine mathematical ideas more closely and, consequently, to deepen her understanding of these ideas. Russell et al. (1995) found that such preparatory exploration caused teachers to consider and see explicitly what they knew implicitly. The process of reading students as they struggled with particular tasks led Jackie and Catherine to explore the mathematical ideas underlying tasks. Similarly, Russell et al. found that episodes involving student confusion or problematic thinking caused teachers to look at related mathematical structures underneath students' reasoning. In essence, students' difficulties

became a catalyst for teachers' further exploration of the topic. Finally, the type of mathematics learning both teachers experienced when reading tasks students were engaged in is similar to learning that Russell et al. observed when teachers engaged in mathematical thought with students. They found that examining students' responses with respect to the demands of a task led teachers to "think through the mathematics again for themselves in new ways, seeing new aspects of familiar content" (p. 3). Similarly, Catherine and Jackie found that analyzing a task in order to make sense of students' responses to it helped them see mathematics differently.

The two teachers' reading activities involved another type of learning that appeared to play a significant role in the changes observed in their practices. Both teachers learned about students. Through observing and analyzing their approaches to and struggles with mathematical tasks, the teachers gained insights into students' abilities and ways of thinking and learning. Their learning from encounters with their students' understandings and struggles prompted teachers to refine their goals and expectations for students' learning. These changes seemed to influence their interpretations of and responses to students' work.

Teacher Learning and the Textbook

What role did the textbook play in the teachers' learning? The above analysis reveals two characteristics of the text's role. First, the most fruitful learning occurred when the teachers read the text, their students, or tasks as students worked on them to make curricular decisions. This process of reading and decision making caused the teachers to reexamine their beliefs and understandings, which, in turn, influenced the curriculum they enacted. Second, the teachers were more likely to engage in this type of decision making when the text was less directly involved in the curriculum process. Substantial learning, for example, occurred for both teachers in the process of enacting tasks in the classroom. During this process, the text's role was least direct. That is, the classroom activities may have been initiated by tasks or ideas from the text, but it was the teachers who were actively involved in reading and acting on these events. The instances that prompted less learning were those in which a teacher used suggestions in the text verbatim, without needing to make adaptations or respond to unexpected student thinking, or when she completely dismissed parts of the text. It is probable that teachers regularly learn from reading students and tasks, regardless of the source of the tasks. However, these teachers faced more unanticipated student responses or struggles when they drew unfamiliar activities or mathematical ideas from the text than when they relied on familiar routines.

These characteristics of the text's role in the teachers' learning are reminiscent of McLaughlin's (1976) discussion of mutual adaptation in instructional policy initiatives. She found that effective implementation occurred when teachers and administrators adapted the project design to their circumstances as well as to the local conditions. This process involved making goals and assumptions explicit and fostered learning-by-doing. In other words, through the adaptation process, both the school personnel and the programs they had set out to implement changed.

The HBJ textbook contributed to Catherine's and Jackie's learning in a similar way. The textbook provided two types of resources that the teachers used in enacting curriculum that led to their learning: it proposed the mathematical terrain within which the teachers constructed curriculum, and it provided materials with which to build. The mathematical terrain defined a set of topics to be addressed. The building materials included mathematical concepts, representations, and tasks. By themselves, these resources did not provoke reflection or change. Adapting them to their students led to teacher learning. Catherine, for example, used unfamiliar tasks from the text

that alone did not motivate her to rethink existing ideas. The tasks gave rise to episodes in her teaching that required her to read students and tasks in order to make decisions. Jackie's learning began when she examined the suggestions in the text in order to design her own tasks. It continued as she analyzed and responded to students as they worked on these tasks. In other words, the growth that both teachers experienced occurred when they engaged in pedagogical reading and decision making.

Ironically, the HBJ text developers did not intend to engage teachers in reading and decision making; instead, the developers offered a set of prepared activities, within a specific structure, for teachers to use. Though the vast amount of material in the text made it necessary for teachers to choose from among tasks, the text did not explicitly acknowledge a need for such selection. Nor did it refer to other dimensions of pedagogical decision making. (See Remillard, 1996, for detailed analysis of the HBJ textbook.)

Conclusion and Implications

The questions underlying this study were whether and how redesigned curriculum materials might support curricular and pedagogical change in mathematics education. My findings suggest that texts might contribute to change in teaching, but through routes not commonly considered. The assumption often motivating the design and selection of reform-oriented curriculum materials is that textbooks have the potential to shape the enacted curriculum (Tyson-Bernstein & Woodward, 1991). In this view, the job of curriculum writers is to provide teachers with reform-based activities that they can "implement" in their classrooms (Russell, 1994). The variation in the two teachers' readings of the text, however, illustrates the powerful role that teachers play in mediating the textbook's contribution to the enacted curriculum. Consequently, it is unlikely that textbooks can shape the curriculum directly. The centrality of reading in the teachers' learning, however, suggests that texts might contribute more effectively to the enacted curriculum by fostering teachers' reading and subsequent decision making.

By drawing on the analysis of Jackie's and Catherine's teaching, one can speculate about the ways that reform-oriented curriculum materials might be designed to support teachers' reading and decision making. I discuss several possibilities below. I use the terms "textbook" and "curriculum materials" interchangeably. Textbooks generally refer to curriculum programs sold by commercial publishing companies that include student texts and a teacher's guide. The term "curriculum materials" has been used in recent years, often by noncommercial curriculum developers, to refer to programs that are unlike textbooks in design and content. The following recommendations apply to all curriculum programs.

What Textbooks Offer Teachers

Mathematics textbooks generally provide resources for two aspects of mathematics instruction: organizing the mathematical content and teaching the content (Remillard, 1996). The HBJ textbook attended to both aspects, but as I discussed earlier, not necessarily in ways designed to engage teachers in curriculum enactment. The focus of the text's offerings was on the tasks to give students and the order in which to give them. The text did not provide rationales for the content of each lesson or for the sequencing. Nor did it discuss how teachers might make sense of or use new activities that were obviously inspired by reform initiatives. The text's offerings were intended to be a finished curriculum for teachers to enact.

Catherine's learning from selecting tasks in the text provides evidence that well-developed activities might contribute to changes in teachers' ideas about mathematics learning. Through offering tasks, the text introduced new ideas into her practice; she examined them, decided how to act on

them, and came to see them as valuable. In this way, curriculum materials have the potential to foster teachers' learning through providing a collection of nonstandard tasks for teachers to try. This type of resource, however, is not sufficient. Catherine's struggles with many of the new tasks, together with her frustration that the teacher's guide did not provide her with the pedagogical guidance to use these tasks, illustrated the limitations of the textbook's approach. "The book doesn't go into a lot of ideas on how to get the children to come to this conclusion" (Interview, April 3, 1993). To support teachers' learning from these resources, curriculum materials need to assist teachers in interpreting and using tasks.

Similarly, Jackie's learning from reading ideas in the text and inventing tasks illustrates how examining mathematical ideas in preparation for teaching them can enhance teachers' mathematical understandings. Thus, curriculum materials that provide teachers like Jackie with plenty of resources to build with can foster teachers' learning from inventing tasks. Jackie viewed the text as a source of ideas. "It was at least my jumping-off point and it did, at least, start me thinking" (Interview, July 1, 1993). In contrast, Jackie's tendency to dismiss ideas in the text that she did not find immediately appealing, such as estimation, suggests that the text might need to offer more in the way of explanation and rationale for many of its unfamiliar suggestions.

Teachers reading the text. To make a greater contribution to teachers' reading of texts, text writers must offer more than tasks. They need to provide information that will help teachers assess and interpret specific tasks. For example, in addition to describing a task, teachers' guides might discuss the underlying mathematical goals, suggesting ways that the task might be made more or less complex but still focused on the intended goal. They might also suggest alternative tasks or approaches teachers have used to teach a mathematical idea along with sample student work or responses. Because teachers might attend to different parts of a text, writers will need to describe how the different components are related to one another and to the overall goals of a lesson or unit.

Teachers' reading of students and tasks. To contribute to teachers' reading of students and tasks, texts must prompt teachers to examine students' thinking with respect to the goals of mathematical tasks. This sort of reading might be encouraged through descriptions of possible student responses or work and how they could be interpreted. Researchers have found that examining students' invented solution strategies to problems can pique teachers' curiosity about their own students' approaches (Carpenter et al., 1988; Davenport & Sassi, 1995; Featherstone & Smith, 1996). The rapidly growing body of research on children's thinking in mathematical domains can contribute to frameworks for understanding students' reasoning. For example, researchers involved in the Cognitively Guided Instruction projects have developed frameworks for students' thinking about addition, subtraction, multiplication, division, and base-10 knowledge (Carpenter, Fennema, & Franke, 1994). Others have examined students' understandings and abilities with respect to the concepts of ratio and proportion (Mack, 1990; Parker & Leinhardt, 1995), estimation and number sense (Sowder, 1992), geometry and spatial reasoning (Clements & Battista, 1992), and probability and statistics (Shaughnessy, 1992). By drawing on studies such as these, texts can offer teachers lenses for examining students' understandings.

Teachers' guides can also focus teachers' reading on the processes through which students develop understandings during classroom discourse by providing images of classroom interaction. Alternative curriculum materials developed in the past several years provide samples of discourse that might occur as a class pursues a task. These can provide "useful images of teachers engaging students in mathematical investiga-

tions [that] allow teachers to envision the taking on of similar actions" (Davenport & Sassi, 1995, p. 40). In addition to providing images of student discourse, teachers' guides might assist teachers in the type of reading Jackie and Catherine found themselves doing in two additional ways. First, they might use annotations to provide the reader access to the teachers' thinking during the interaction. Sample dialogue that leaves teachers with images of rich classroom discourse runs the risk of presenting the finished interaction and not the analysis that guided the teacher's orchestration of it. Second, sample dialogues could also include examples in which students or teachers are genuinely perplexed about how to proceed, along with commentary on how these struggles might be assessed. Again, such details could emphasize the reading process rather than focus solely on implementation.

Teachers' decision making. The other curriculum enactment activity that reform-oriented textbooks need to attend to is decision making. The purpose of both teachers' reading was to determine how to act on suggestions in the text and events in their teaching. In order to contribute to teachers' decision making, text writers need to build into their materials spaces and support for teachers to make decisions. My use of the term "space" has two connotations. It refers to a measure of flexibility in which teachers can maneuver, as well as the room to work. Texts designed to guide teachers' decision making need to include both kinds of space; they need to be flexible and responsive to teachers' choices as well as incomplete without teachers' input. Ironically, the idea that space should be added to curriculum materials runs counter to many text writers' inclinations. Every possible space in the teacher's guide, for instance, was overflowing with activities to give students. In order to provide space for teachers' decision making, texts would need to be much less "finished" than they currently are.

The idea that curriculum materials might make spaces for teacher decision making is similar to Bridgham's (1971) view that good curriculum materials might contain multiple possible routes through a defined pedagogic terrain. Much like selecting among the many routes through a city, the paths that teachers and students take through the pedagogic space are not predetermined by writers but are the results of day-to-day and moment-to-moment decisions. The image of curriculum materials as offering flexibility of movement is similar to Shulman's (1983) conception of educational policy. He suggests that policy should be "designed as a shell within which the kernel of professional judgment and decision making can function comfortably" (p. 501). Bridgham's idea that the curriculum contains possible routes through particular terrain, however, distinguishes the role of curriculum materials from that of policies in a crucial way; in addition to offering freedom of movement within pedagogic space, curriculum materials must also assist teachers in navigating through that space productively. As Heaton (1994) put it, "Curriculum developers must find ways to guide teachers' pedagogical and mathematical decisions, not make decontextualized decisions for them" (p. 376).

Who Textbooks Are For

Underlying these recommendations is the stance that the audience of curriculum materials is teachers rather than students. Naturally, teachers' guides are written for teachers, but my analysis of the HBJ text as a whole suggested that its main focus was on shaping students' experiences in the mathematics classroom (Remillard, 1996). Its communication with teachers was wholly to this end. Thus, it was written with the activities of students, rather than teachers, in mind. The implicit role of teachers was to "deliver" activities to students. Such a stance conflicts with the goals of the reforms.

Central to the reform agenda is the idea that students' understanding of mathemat-

ics should enable them to reason and solve problems rather than merely to follow rules or memorize facts. To this end, reformers have emphasized activities in which students solve problems and discuss their ideas with others. Such an instructional format involves a significant shift from those found in commercial textbooks in which the teacher or textbook poses questions that students answer. Others have noted that mathematics teaching envisioned by current reforms involves student activities such as sustained work on complex and demanding tasks (Stein, Grover, & Henningsen, 1996) and discourse with peers (Ball, 1997). It is unlikely that textbooks can bypass the teacher in the process of orchestrating such activities. Furthermore, helping students learn to think mathematically requires that teachers understand the mathematics themselves as well as attend to students' developing understandings to determine what to pursue in a lesson.

If curriculum materials are to support teachers' efforts to learn to teach differently, they must be written for teachers as readers. Like the majority of teachers' guides, the HBJ guide was designed to provide teachers with a collection of tasks to give to students. It communicated by speaking *through* teachers, by guiding their actions. It did not speak *to* them about these tasks or the ideas underlying them. This choice of language is common among many curriculum guides, which tend to offer steps to follow, problems to give, actual questions to ask, and answers to expect. This approach to guiding teaching emphasizes the outcomes of teaching and not the rationales, assumptions, or agendas supporting them, discouraging teachers from engaging the ideas underlying the writers' decisions and suggestions.

Guides designed to engage teachers in helpful dialogue need to communicate in a different language in order to attend to the activities of reading and decision making. They must be designed to speak *to* teachers, not merely *through* them. In addition to suggesting to teachers what they might say or do, text writers need to talk to teachers about these suggestions, about the mathematical and pedagogical ideas underlying them, and about students' likely reactions to them. In doing so, they need to make their agendas and perspectives accessible to teachers.

Implications

Teacher development. My findings and subsequent recommendations for revised curriculum guides have implications for the education and development of teachers as well. Because the type of interactive relation envisioned between teachers and texts is foreign to most teachers, they are likely to need opportunities to learn to use texts in constructing curriculum. Moreover, as the differences in the ways Catherine and Jackie read the textbook suggest, there is much that revised textbooks cannot accomplish alone. Thus, it is unlikely that reformed curriculum materials will significantly affect teaching if consideration is not given to the nature of support that accompanies them.

Professional development activities and preservice teacher education need to help teachers become more active in the curriculum development process. It is critical, however, that efforts in material development and teacher development are coordinated and coherent. Too often these two components of support for teachers occur at cross-purposes to one another. In fact, many perspectives on teacher-text relationships place the two in opposition, each vying for the same authority over curricular decisions (Ball & Feiman-Nemser, 1988; Russell, 1994). In order to promote productive use of curriculum materials, professional development opportunities need to foster teachers' reading and decision making, deepen and broaden their mathematical knowledge, and develop their knowledge of the workings of the curriculum development industry. Development in these three areas can enhance teachers' abilities to assess students, mathematical tasks, and goals and to critically examine the offerings in their text-

books in order to adapt and transform them.

Context of teaching. The suggestions above all have implications for the structures in which teaching takes place. Curriculum materials and teacher development opportunities do not operate in isolation; their effect on teaching is mediated by the teaching context. Many school structures explicitly and implicitly impose demands on teachers that influence their choices (Hargreaves, 1994; Jackson, 1986; Sarason, 1982). Jackie's and Catherine's cases illustrate how features of the teaching context can affect teachers' efforts to change. Consequently, the adoption of reform-oriented textbooks must occur within broader school and district structural changes that support teachers as curriculum developers rather than as implementers.

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